Activity description

This activity shows students how to use Kruskal's and Prim's algorithms to solve minimum connector problems. A cable TV problem introduces the topic and the rules for the two algorithms. The students are then set a second problem involving a theme park.

Suitability

Level 3 (Advanced)

Time

1–2 hours

Resources

Student information sheet, worksheet, slides

Equipment

Calculators

Key mathematical language

Algorithm, spanning tree, cycle, network, vertex, edge, weight, minimum connector, adjacency matrix

Notes on the activity

This activity can be used to introduce Kruskal's and Prim's algorithms for finding minimum connectors, or used at the end of the course as a revision exercise.

The accompanying slideshow includes the same examples. This and the student sheets can be used as follows.

First use the student information sheet and first three slides to introduce the context. Explain that the cable TV company intends to run its cables alongside the existing roads (shown in red on the map), and that the table at the bottom of the information sheet gives the distances in miles.

Ask students (individually or in pairs) to find one way of connecting all the towns on the map, trying to keep the total length of the connecting edges as low as possible. They can use the map on the student information sheet to do this, transferring the lengths they need from the mileage chart.

Discuss the results (noting the minimum length of cable used), and the need for a systematic way of tackling such problems when they involve complex networks.

Use the fourth slide to outline Kruskal's algorithm. If necessary introduce or remind students of related vocabulary such as network, vertex, edge, weight, cycle, spanning tree, and minimum connector.

The fifth and sixth slides show the solution to the cable TV problem – you could use this to demonstrate the method or to check students' solutions after they have tried this themselves using page 2 of the student sheets.

The solution shows that the minimum possible total length of cable is 53.3 miles.

Discuss possible advantages and disadvantages that the solution may have in the real context. For example, a possible disadvantage is that the routes chosen may include some difficult terrain.

Prim's algorithm is outlined on the seventh slide. Students can be asked to do this using the copy of the Isle of Man map and/or the adjacency matrix on page 3 of the student sheets.

Slides nine to eleven give the matrix solution then the resulting network – again this can be used for demonstrating the method or checking students' solutions. Discuss the advantage of using a matrix – this means that Prim's algorithm can be carried out using a computer.

Compare and discuss the two algorithms before asking students to use one or both of them to solve the Theme Park Paths problem.

During the activity

Students can work on the activities in pairs or small groups, and their solutions shared with the whole group.

Points for discussion

Ask students how many edges there will be in a minimum spanning tree that connects n vertices.

Also ask them to explain how the matrix method works for Prim's algorithm.

Encourage students to compare the two algorithms and to think about the advantages or disadvantages that the solutions to the two problems may have in the real world.

Extensions

Further work on minimum connector problems can be found at
www.mymaths.co.uk MyMaths website, requires subscription

http://www.mei.org.uk/ MEI website, requires subscription

http://www-b2.is.tokushima-u.ac.jp/~ikeda/suuri/dijkstra/Prim.shtml

(graphical demonstrations of Prim's algorithm)

http://www-b2.is.tokushima-u.ac.jp/~ikeda/suuri/kruskal/Kruskal.shtml

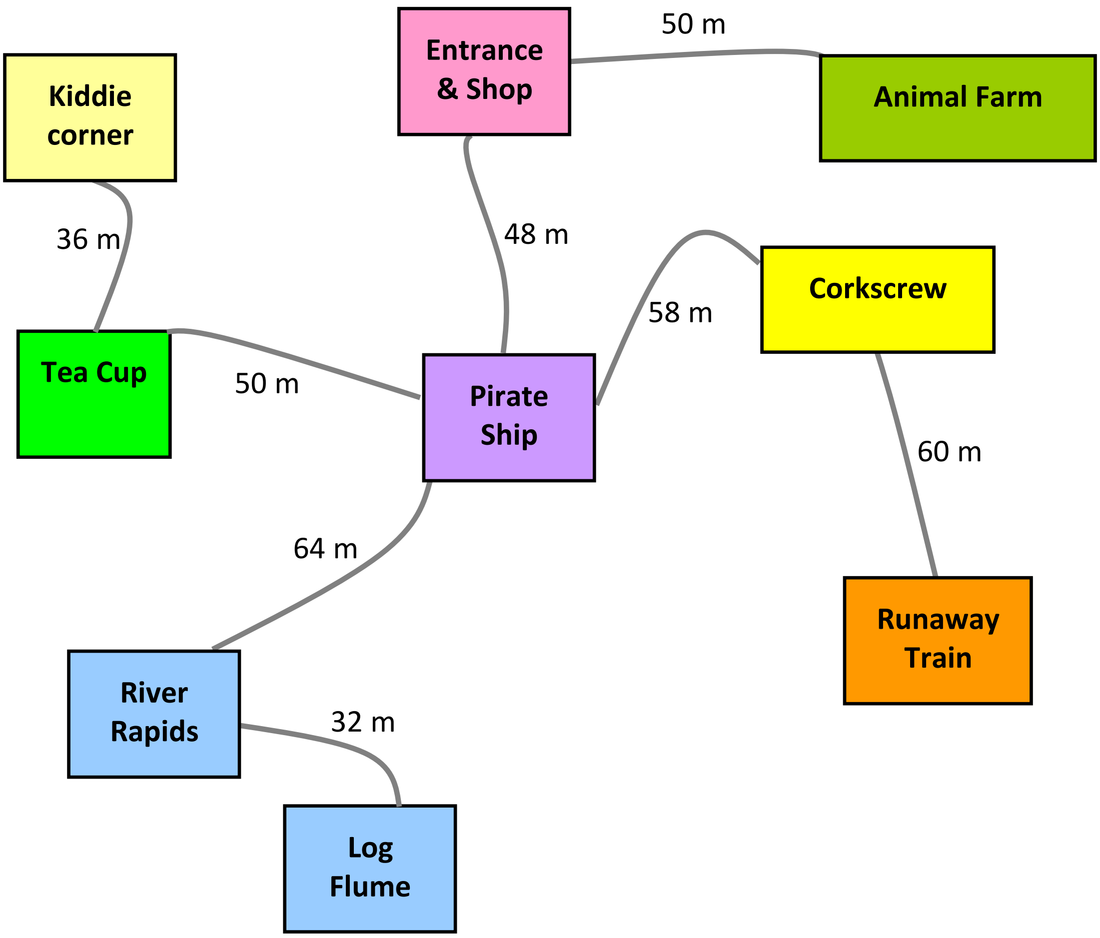
(graphical demonstrations of Kruskal's algorithm)

Answers: Cable TV

Slides 5 and 6 in the slideshow give the solutions to the Cable TV problem using Kruskal's algorithm.

Slides 9 to 11 give the matrix solution and the resulting network for the Cable TV problem using Prim's algorithm.

Answers: Theme Park paths



Using Kruskal's

River Rapids to Log Flume	32
Kiddie Corner to Tea Cup	36
Entrance to Pirate Ship	48
Entrance to Animal Farm	50
Tea Cup to Pirate Ship	50
Pirate Ship to Corkscrew	58
Corkscrew to Runaway Train	60
Pirate Ship to River Rapids	64

either way round (the two 50s)

The total length of pathway in the minimum connector
= 32 + 36 + 48 + 50 + 50 + 58 + 60 + 64 = **398 metres**

Answers: Using Prim's

	1	3	5	6	4	2	8	7	9
	E	**A**	**K**	**C**	**T**	**P**	**RR**	**RT**	**L**
E	-	50	54	-	-	48	-	-	-
A	50	-	-	-	-	75	-	120	-
K	54	-	-	-	36	75		-	-
C	-	-	-	-	-	58	-	60	150
T	-	-	36	-	-	50	-	-	-
P	48	75	75	58	50	-	64	-	72
RR	-	-	-	-	-	64	-	-	32
RT	-	120	-	60	-	-	-	-	80
L	-	-	-	150	-	72	32	80	-

Key

E Entrance & Shop
A Animal Farm
K Kiddie Corner
C Corkscrew
T Tea Cup
P Pirate Ship
RR River Rapids
RT Runaway Train
L Log Flume

Entrance to Pirate Ship	48
Entrance to Animal Farm	50
Pirate Ship to Tea Cup	50
Tea Cup to Kiddie Corner	36
Pirate Ship to Corkscrew	58
Corkscrew to Runaway Train	60
Pirate Ship to River Rapids	64
River Rapids to Log Flume	32

either way round

The total length of pathway in the minimum connector

= 48 + 50 + 50 + 36 + 58 + 60 + 64 + 32 = 398 metres